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Testing the direct CP violation of the Standard Model without knowing strong phases

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Abstract

The strong phase of a two-body decay amplitude of a heavy particle depends on decay operators even if the final state is an eigenstate of isospin or SU(3) quantum numbers. This particular property extends the opportunity of testing consistency of experimentally observed CP -violation phases with the Standard Model without knowing strong interaction effects in decay amplitudes. With three generations, the Standard Model requires $\Delta(\pi^\pm\eta') = -\Delta(K^\pm\eta')$ in the flavor SU(3) symmetry limit, where $\Delta(\pi^\pm\eta') \equiv \text{B}(B^+ \rightarrow \pi^+\eta') - \text{B}(B^- \rightarrow \pi^-\eta')$ and $\Delta(K^\pm\eta') \equiv \text{B}(B^+ \rightarrow K^+\eta') - \text{B}(B^- \rightarrow K^-\eta')$. However, testing this relation with the Standard Model is not easy. The relation $\Delta(\pi^\pm\psi) = -\Delta(K^\pm\psi)$ is cleaner but even harder to test.

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I. INTRODUCTION

In the minimal Standard Model with three generations (MSM) there is only one independent CP -violating parameter. Therefore, in principle, determining the weak phase β from the CP violating $B^0 - \bar{B}^0$ mixing is sufficient within the MSM. One of the major purposes of exploring for the phase γ in direct CP violation processes is to test consistency of other CP violation phenomena with the MSM and to search for possible sources of CP violation beyond the MSM.

Many proposals have been made as to how to extract the phase γ from direct CP -violating processes [1]. The difficulty is that the weak phases are entangled with unknown strong phases due to final state interactions (FSI). In many cases, one can in principle determine both weak and strong phases by measuring sufficiently many decay modes. Since experimental errors accumulate with the number of measured values, however, an unrealistically high precision is often required for measurement. Use of flavor SU(3) symmetry is a powerful way to simplify the theoretical analysis by reducing the number of independent decay amplitudes. Nevertheless, additional dynamical approximations and/or assumptions are needed to make the extraction of γ feasible. While a model such as the factorization model may give us some idea of the relative magnitudes of decay amplitudes, the strong phases of amplitudes are much harder to compute unless short-distance QCD should completely dominate.¹ Because of the uncertainty of strong phases some are content only with setting bounds on the phase γ .

In order to extract the weak phases from direct CP violations, we need a set of decay modes which are described by two or more of independent decay amplitudes differing in the strong phase. The fewer the independent amplitudes are, the simpler the analysis is. We would like to avoid theoretical assumptions and approximations on those decay amplitudes as much as possible, preferably treating them as free parameters without theoretical prejudice. For this reason, we should study a set of decay modes that involves the smallest number of independent amplitudes. With SU(3) symmetry He recently derived several relations [4] for the rate differences of the two-body octet pseudoscalar-meson decay modes which do not depend on strong interaction effects at all. The final states considered by He contain two or more of isospin or SU(3) eigenstates to generate a strong phase difference. However, high inelasticity and multichannel coupling of the final states of the B decay make a CP asymmetry observable even in the final states which are eigenstates of isospin or SU(3). We shall briefly remind this important fact in Section II in order to add a few more promising relations of the same nature to the list of [4]. In Section III, we derive the relation for the rate differences of singlet-octet two-body final states, which are not only isospin eigenstates but also octet eigenstates of SU(3). Comments will be made on feasibility of test in Section IV.

¹If the strong phases of two-body B decay are dominated by short-distance QCD [2,3], all strong phases would be small and calculable in principle. However, a convincing *quantitative* proof is yet to be given for the short-distance dominance.

II. FINAL STATE INTERACTION

When many decay channels are open in a heavy particle decay, the FSI phases of decay amplitudes for experimentally measured final-states are not simply related to the phases of pure strong interaction. Take, for example, a two-body final state $|ab\rangle$. The state $|ab\rangle$ is not one of the eigenstates $|\alpha\rangle$ of strong interaction S matrix. When the eigenchannels of S matrix are defined by

$$\begin{aligned}\langle\beta|S|\alpha\rangle &= \langle\beta^{\text{out}}|\alpha^{\text{in}}\rangle, \\ &= \delta_{\beta\alpha}e^{2i\delta_\alpha},\end{aligned}\tag{1}$$

an experimentally observable final-state is a linear combination of them. Take, for example, a two-body final state $|ab\rangle$. The state $|ab\rangle$ is expanded as

$$|ab\rangle = \sum_{\alpha} O_{ab,\alpha}|\alpha\rangle.\tag{2}$$

Time reversal invariance of strong interaction allows us to choose the S matrix to be symmetric and $O_{ab,\alpha}$ to be an orthogonal matrix. For a CP -even decay operator \mathcal{O}_i , time-reversal operation leads us to

$$\langle\alpha^{\text{out}}|\mathcal{O}_i|B\rangle = \langle B|\mathcal{O}_i|\alpha^{\text{out}}\rangle\langle\alpha^{\text{out}}|\alpha^{\text{in}}\rangle.\tag{3}$$

Therefore the decay amplitude takes the form

$$\langle\alpha^{\text{out}}|\mathcal{O}_i|B\rangle = M_{\alpha}^i e^{i\delta_{\alpha}},\tag{4}$$

where

$$(M_{\alpha}^i)^* = M_{\alpha}^i.\tag{5}$$

This is the well-known phase theorem [5] in the case that the final state is an eigenstate of S matrix. When $|ab\rangle$ is not an eigenstate of S matrix, but is given by Eq. (2), the decay amplitude for $B \rightarrow ab$ is a superposition of $B \rightarrow \alpha$:

$$M^i(B \rightarrow ab) = \sum_{\alpha} O_{ab,\alpha} e^{i\delta_{\alpha}} M_{\alpha}^i.\tag{6}$$

We should learn two important facts from Eq. (6). One is that the net (strong) phase of $M(B \rightarrow ab)$ is not simply related to the eigenphase shifts δ_{α} of S matrix. It is not given by a phase of any pure strong interaction process, elastic or inelastic, of $|ab\rangle$. The other is that the phase of $M(B \rightarrow ab)$ is dependent on the operator \mathcal{O}_i . For instance, the strong phase of the $B \rightarrow K\pi$ amplitude into total isospin 1/2 takes different values for the tree decay process and for the penguin decay process. There is no reason to expect that the two values are even close to each other, since the different quark structures of $\mathcal{O}_{1,2}$ and $\mathcal{O}_{3\sim 10}$ generate very different sets of M_{α}^i in general. The strong phases of the tree and the penguin amplitude of $(K\pi)_{I=1/2}$ can be just as much different as those of $(K\pi)_{I=1/2}$ and $(K\pi)_{I=3/2}$ are, or as those of $(K\pi)_{\mathbf{8}}$ and $(K\pi)_{\mathbf{27}}$ of $SU(3)$ are.

Thanks to this property of the FSI in the B decay, the CP asymmetry can appear even in an isospin eigenstate or an $SU(3)$ eigenstate. A merit of considering such final states is that since their strong interaction parametrization is very simple, we can more easily disentangle the weak phases from the strong phases.

III. SU(3) ANALYSIS

We cast the effective Hamiltonian of the B decay into the form

$$H_{eff} \simeq 2\sqrt{2}G_F \sum_{q=d,s} (V_{ub}V_{uq}^* \sum_{i=1}^2 C_i \mathcal{O}_i^q - V_{tb}V_{tq}^* \sum_{j=3}^{10} C_j \mathcal{O}_j^q) + \text{H.c.}, \quad (7)$$

where the decay operators are defined by

$$\mathcal{O}_1^q = (\bar{u}\gamma^\mu b_L)(\bar{q}\gamma_\mu u_L) - (\bar{c}\gamma^\mu b_L)(\bar{q}\gamma_\mu c_L), \quad (8)$$

$$\mathcal{O}_2^q = (\bar{q}\gamma^\mu b_L)(\bar{u}\gamma_\mu u_L) - (\bar{q}\gamma^\mu b_L)(\bar{c}\gamma_\mu c_L), \quad (9)$$

$$\mathcal{O}_3^q = \sum_{q'=u,d,s,c} (\bar{q}\gamma^\mu b_L)(\bar{q}'\gamma_\mu q'_L) + \frac{C_2}{C_3}(\bar{q}\gamma^\mu b_L)(\bar{c}\gamma_\mu c_L), \quad (10)$$

$$\mathcal{O}_4^q = \sum_{q'=u,d,s,c} (\bar{q}_\alpha\gamma^\mu b_{\beta L})(\bar{q}'_\beta\gamma_\mu q'_{\alpha L}) + \frac{C_1}{C_4}(\bar{q}_\alpha\gamma^\mu b_{\beta L})(\bar{c}_\beta\gamma_\mu c_{\alpha L}), \quad (11)$$

$$\mathcal{O}_5^q = \sum_{q'=u,d,s,c} (\bar{q}\gamma^\mu b_L)(\bar{q}'\gamma_\mu q'_R), \quad (12)$$

$$\mathcal{O}_6^q = \sum_{q'=u,d,s,c} (\bar{q}_\alpha\gamma^\mu b_{\beta L})(\bar{q}'_\beta\gamma_\mu q'_{\alpha R}), \quad (13)$$

$$\mathcal{O}_7^q = \frac{3}{2} \sum_{q'=u,d,s,c} (\bar{q}\gamma^\mu b_L)e_{q'}(\bar{q}'\gamma_\mu q'_R), \quad (14)$$

$$\mathcal{O}_8^q = \frac{3}{2} \sum_{q'=u,d,s,c} (\bar{q}_\alpha\gamma^\mu b_{\beta L})e_{q'}(\bar{q}'_\beta\gamma_\mu q'_{\alpha R}), \quad (15)$$

$$\mathcal{O}_9^q = \frac{3}{2} \sum_{q'=u,d,s,c} (\bar{q}\gamma^\mu b_L)e_{q'}(\bar{q}'\gamma_\mu q'_L), \quad (16)$$

$$\mathcal{O}_{10}^q = \frac{3}{2} \sum_{q'=u,d,s,c} (\bar{q}_\alpha\gamma^\mu b_{\beta L})e_{q'}(\bar{q}'_\beta\gamma_\mu q'_{\alpha L}). \quad (17)$$

In grouping the terms in H_{eff} , we have expressed the coefficient $V_{cb}V_{cq}^*$ of the tree operators involving c and \bar{c} in terms of $V_{ub}V_{uq}^*$ and $V_{tb}V_{tq}^*$ by using the unitarity relations of three generations,

$$V_{ub}V_{uq}^* + V_{cb}V_{cq}^* + V_{tb}V_{tq}^* = 0, \quad (q = d, s), \quad (18)$$

and have distributed them into $\mathcal{O}_{1\sim 4}$ in Eq. (8) \sim (11). The tree operators of $c\bar{c}$ are potentially important if the FSI should allow a substantial conversion of $c\bar{c} \rightarrow$ light quark pairs [6].

It is important to notice here that all decay operators ($\mathcal{O}_i^d, \mathcal{O}_i^s$) ($i = 1 \sim 10$) form doublets under the U -spin rotation ($d \leftrightarrow s$) of an SU(3) subgroup. Under U -spin, B^\pm are singlets while (B^0, \bar{B}_s^0) forms a doublet. Likewise (π^-, K^-) is a doublet.² Here we consider the B^\pm decay into $\pi^\pm\eta'$ and $K^\pm\eta'$ instead of the B^0/\bar{B}^0 and B_s/\bar{B}_s decays:

²This U -spin property immediately leads to six of the relations written in [4]

$$B^\pm \rightarrow K^\pm \eta', \quad (19)$$

$$B^\pm \rightarrow \pi^\pm \eta'. \quad (20)$$

In the SU(3) symmetry limit leaving out the $\eta - \eta'$ mixing, the decay amplitudes for $B^\pm \rightarrow \pi^\pm \eta'$ and $K^\pm \eta'$ are parametrized in the form

$$M(\pi^+ \eta') = V_{ud} V_{ub}^* T + V_{td} V_{tb}^* P, \quad (21)$$

$$M(K^+ \eta') = V_{us} V_{ub}^* T + V_{ts} V_{tb}^* P, \quad (22)$$

where

$$T = 2\sqrt{2}G_F \langle \pi^+ \eta' | \sum_{i=1}^2 C_i \mathcal{O}_i^\dagger | B^+ \rangle \quad (23)$$

$$P = 2\sqrt{2}G_F \langle \pi^+ \eta' | \sum_{j=3}^{10} C_j \mathcal{O}_j^\dagger | B^+ \rangle. \quad (24)$$

The QCD and electroweak penguin contributions have been combined into a single term

$$P = P_{QCD} + P_{EW}. \quad (25)$$

The decay amplitudes for $B^- \rightarrow \pi^- \eta'$ and $K^- \eta'$ are obtained from Eqs. (21) and (22) by complex conjugation of the quark mixing matrix elements.

The FSI turns the amplitudes T and B complex and, according to our argument in Section II, their phases are different from each other in general. Therefore the rate differences

$$\Delta(\pi^\pm \eta' (K^\pm \eta')) = B(B^+ \rightarrow \pi^+ \eta' (K^+ \eta')) - B(B^- \rightarrow \pi^- \eta' (K^- \eta')) \quad (26)$$

$$= 4|T||P| \sin \delta\theta \operatorname{Im}(V_{uq} V_{ub}^* V_{tb} V_{tq}^*) \quad (q = d, s), \quad (27)$$

where $\delta\theta = \arg(T^*P)$, are nonvanishing. Though the final states are isospin eigenstates, $\Delta(\pi^\pm \eta')$ and $\Delta(K^\pm \eta')$ can be just as large as those of isospin non-eigenstates. The imaginary part of the product of the quark mixing matrix elements is common to $q = d$ and s up to a sign [7]:

$$\operatorname{Im}(V_{ud} V_{ub}^* V_{tb} V_{td}) = -\operatorname{Im}(V_{us} V_{ub}^* V_{tb} V_{ts}). \quad (28)$$

We thus come to the relation,

$$\Delta(\pi^\pm \eta') = -\Delta(K^\pm \eta'). \quad (29)$$

This relation is not useful in extracting the weak phase γ unless we know $|T||P|$ and $\delta\theta$ beforehand from somewhere else. From the viewpoint of testing CP violations in the MSM, however, it is one of the cleaner tests and will serve the same goal as determining γ through complex procedures.

IV. COMMENTS ON SU(3) BREAKING

The $K^\pm\eta'$ mode is the largest in branching fraction among all charmless two-body B^\pm decay modes so far measured. The $\pi^\pm\eta'$ mode has not been measured. In a theoretical analysis based on SU(3) [8], $\pi^\pm\eta'$ is expected to be competitive with $\pi^\pm\pi^0$ and to be one of the largest in branching fraction among the flavorless final states. Measurability of a CP asymmetry in $\pi^\pm\eta'$ was actually pointed out by the authors of [8] and [9]. The competitive rates of $K^\pm\eta'$ and $\pi^\pm\eta'$ may give an advantage to Eq. (29) over the relation $\Delta(\pi^+K^0/\pi^-\bar{K}^0) = -\Delta(K^+\bar{K}^0/K^-K^0)$ of [4].

We have ignored SU(3) breaking of strong interaction in Eq. (29). It is likely that the SU(3) breaking in rescattering dynamics is insignificant at the energy of B mass. In the factorization model, the SU(3) breaking associated with each meson can be incorporated by $\Delta \rightarrow f_{\pi(K)}\Delta$. We shall learn more about reliability of factorization by comparing the theoretical predictions with experiment [8].

The $\eta - \eta'$ mixing is one manifestation of SU(3) breaking. This may be viewed as a disadvantage of our relation. Recently a dynamical model was proposed to compute the decay matrix elements of $B^\pm \rightarrow \pi^\pm\eta'$ and $K^\pm\eta'$ [10]. In this model η' is generated through two gluons in the penguin diagrams while $u\bar{u}$ forms η' in the tree diagrams as a color-favored process. If these processes are the dominant ones, the $\eta - \eta'$ mixing correction appears as a common factor on both sides of Eq. (29) and does not affect the relation.

Since we expect $B(B^\pm \rightarrow K^\pm\eta')$ to be much larger than $B(B^\pm \rightarrow \pi^\pm\eta')$, a small difference between two large numbers will be searched for in the right-hand side of Eq. (29), while the left-hand side will be obtained hopefully as a fairly large difference between two smaller numbers. If we take the estimates by the authors of [10] as a ballpark figure, their preferred values for $B^\pm \rightarrow \pi^\pm\eta'$ lead to $\Delta(\pi^+\eta') - \Delta(\pi^-\eta') \simeq 4 \times 10^{-6}$ which corresponds to a 40% asymmetry. Then we shall be looking for a 3% of asymmetry in the $K^\pm\eta'$ mode up to a possible 22% upward correction due to f_K/f_π . If this is the case, testing the relation with the MSM will be rather a remote possibility in the B factory experiment.

The same relation as Eq. (29) should hold for $B^\pm \rightarrow \rho^\pm\eta'$ and $K^{*\pm}\eta'$:

$$\Delta(\rho^\pm\eta') = -\Delta(K^{*\pm}\eta'). \quad (30)$$

We can replace ρ^\pm and $K^{*\pm}$ with the corresponding components of any meson octet, respectively.

Finally, it is tempting to try for $B^\pm \rightarrow \pi^\pm\psi$ and $K^\pm\psi$

$$\Delta(\pi^\pm\psi) = -\Delta(K^\pm\psi), \quad (31)$$

since the relation is free from the $\eta - \eta'$ mixing contamination. Here again we may replace π^\pm and K^\pm with the corresponding components of any meson octet. Furthermore, the rates are high and the experimental signature of $l^+l^-\pi^\pm(K^\pm)$ is very clean. Unfortunately the asymmetries are will be even smaller.

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